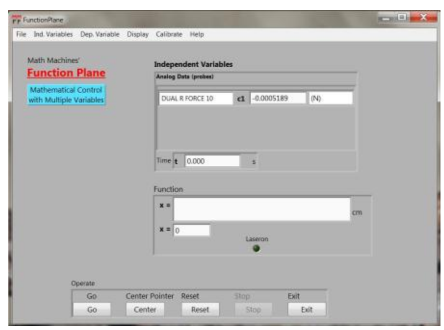
Spring isolator under a three-story town-house in Santa Monica, CA.
Wikimedia Commons
Courtesy of David Ming-Li Lowe

Background:

Robert Hooke was born in England in 1635, and at age 13 moved to London to develop his artistic skills in the studio of the painter Sir Peter Lely. At 18 he was able to take a "poor scholar's place" at Christ Church, Oxford University. He worked as a chemical assistant, eventually employed by Robert Boyle. During his first years there, Hooke developed spiral springs to be used instead of pendulums so time could be told from a pocket watch instead of a clock. This led to a major advancement in navigation since explorers could make accurate time measurements at sea. Hooke was Curator of Experiments for the Royal Society of London for many years. A colleague and competitor of Isaac Newton, he was considered almost as brilliant, with achievements in mechanics, geometry, architecture, astronomy, earthquakes, microscopic studies, and elasticity.

Hooke first published his law for elastic bodies in 1660 at the age of 25. In modern form, the law states: *There is a direct linear relationship between the force applied to a material and the amount it deforms (stretches or compresses).* After 350 years, Hooke's law remains important for understanding springs of all sizes, from those in ball point pens to automobile suspensions to earthquake-resistant building supports. Modified forms of Hooke's law are also important for understanding the behavior of countless objects which look nothing like conventional springs, including the compression of a bat as it strikes a fastball, the stretching of cable supports on a suspension bridge and the bending of steel girders as a skyscraper sways slightly in the wind.



This activity uses a Class II diode laser, similar to those used in many barcode scanners. Never look directly into the laser beam and never allow it to shine into anyone's eyes.

Task: In this activity you will use ratios and rates of change to develop a mathematical model for the change in length of a spring when it is subjected to tension or compression forces, and you will use your model to create a system in which a laser points automatically to the spring's location for any force you apply.

Additional Materials: Support rod, right-angle clamp, C-clamp, spring, force probe, hooked weight with target, "Hooke's Law" overlay.

Math Machines Program:

Function Plane

Activity File: Hooke

Assemble the Function Plane as shown and use a C-clamp to attach it to the table. Plug the Force Sensor into "Ch. 1" on the SensorDAQ BEFORE opening the program "Function Plane." Answer "yes" when the program asks if there are analog probes, and load the Activity File, "Hooke."

Part 1, Using Hooke's Law to Determine a Spring Constant:

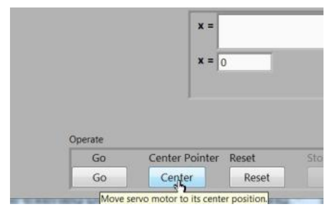
One commonly encountered form of Hooke's Law is the *spring equation*, which relates the force, F, exerted ***on*** a spring to the distance, x, which the spring is stretched or compressed. In this equation, k is the "spring constant," measured in units of force per length.

$$F = kx$$

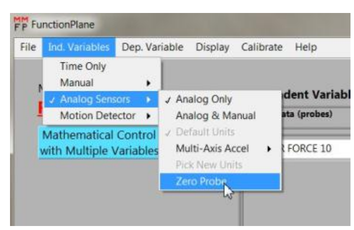
An alternative version of Hooke's Law is "F = -kx," which describes the force exerted ***by*** the spring. This force is in the opposite direction to the displacement, x. It is called a "restoring force", as it tends to restore the system to equilibrium.

It is important to remember that these two forms of expression for Hooke's Law apply ***only if*** the origin has been selected so that zero force corresponds with zero position, making the relationship a true proportion where k is the constant ratio of applied force to the distance the spring is stretched or compressed. A more general form of the law relates *changes* in force, ΔF, to *changes* in position, Δx, regardless of the origin from which F and x are measured.

$$\Delta F = k\,\Delta x$$



1. Hang the hooked weight from the spring to act as a handle and marker. Adjust the force probe's height and horizontal position so the target line on the weight is directly in front of the zero point on the printed scale when the target is not moving and not subjected to any outside forces aside from the gravity and the spring. Press the "Center" button on the control screen. The laser dot should strike the line on the target. If not, readjust the position of the force probe. (Do not change the on-screen center adjust.) When you are satisfied with the alignment, close the "Center Adjust" window. The system should now be in static equilibrium. In the space below, draw a free-body diagram to show the balanced forces acting on the hanging weight.



2. At this point, the Force Probe is displaying the magnitude of the upward force which the Probe applies to the spring, balancing both the weight of the spring and the weight of the object below it. It will be more convenient to have the Probe display *changes* in this force as you move the target weight away from its initial ("zero") position. Use the program's menu selection "<u>Ind. Variables / Analog Sensors / Zero Probe</u>" to make this the zero point for future force measurements. (The probe may show small values that result from vibrations, air movements and other factors, but the readings should stay between +0.01 N and -0.01 N while the target is in initial equilibrium position.) Explain below why it is necessary to zero the force probe in order to use Hooke's Law in the form, "F = kx".

3. Run the program.[1] Use your hand to pull the target down and observe the effect the motion has on the force reading. Answer the questions below:
    a. When you move the target down, does the force you exert on the spring increase or decrease? _______________
    b. What is the direction of the force you exert on the spring? _______________
    c. The Force Probe is showing the increase in the upward force which the probe provides. How is the force displayed on the screen related to the force which you apply? Why?

    d. When you pull the target down, does the length of the spring increase or decrease? __________
    e. Note that the coordinate system on the "Hooke's Law" overlay uses down as the positive direction for both force and position. Is the change in the spring's length, Δx, positive or negative? ________________

4. Use your hand to push the target up and observe the effect the motion has on the force reading. Answer the questions below:
    a. When you move the target up, what is the direction of the force you provide? _______________
    b. Using down as the positive direction for the force you provide, is this force positive or negative? _____________
    c. Do the sign and magnitude of the force you provide match the computer display? ____________
    d. When you push the target up, does the length of the spring increase or decrease? __________
    e. Is the change in the spring's length, Δx, positive or negative? ________________

5. To make a first measurement of the spring constant, k, pull the target down to a specific position, x. Record both x and the corresponding force, F.

    x = ______________ cm F = ______________N

6. From the change in position, use Hooke's Law to determine the spring constant (stiffness coefficient), k. Show your work below.

    k = _____________ N/cm

[1] At this point, there is no function to control the position of the laser, so it should point constantly at the zero position. Running the program, however, updates the force readings more frequently.

7. Using one measurement in an experiment is not a good practice—it could lead to a large error and/ or a wrong conclusion. Use the following table to record more measurements. Use the table below to record additional measurements. Include both positive and negative values for x (extensions and compressions).

Position, x, (cm)	Force, F, (N)

8. Plot a graph of force as a function of extension and draw the best straight line through your data. Find the slope of this line, $\Delta F/\Delta x$.

9. Explain below how the slope of your line is related to the spring constant, k. Consider both the numeric value and the units of k.

10. Write the algebraic function, F = f(x), which allows you to calculate the force for any position.

    F = ____________________

<u>Part 2, Controlling the Laser:</u>

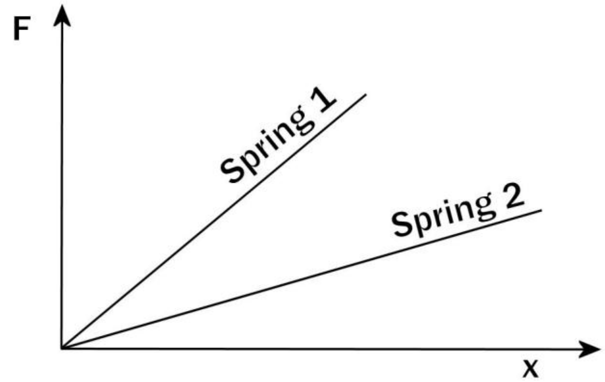
In Part 1, you determined experimentally the force as a function of position. In this part, you will program the computer to calculate and point to the position, x, which corresponds to any force, F.

11. Solve the function you found in step 10 above to find position as a function of force, x = f(F). Enter your function into the Programs function box and run the program. <u>Note that the force, F, is the reading from the SensorDAQ's Channel 1 and MUST be entered as "c1," not "F."</u> Does the laser follow the weight when you lift it up or pull it down? Show your function below and describe in words how it works.

    x = ____________________

Part 3, Using Rates to Determine How Springs Compare:

12. Compare the graphs for the two springs shown at right. What does the slope of the lines represent?

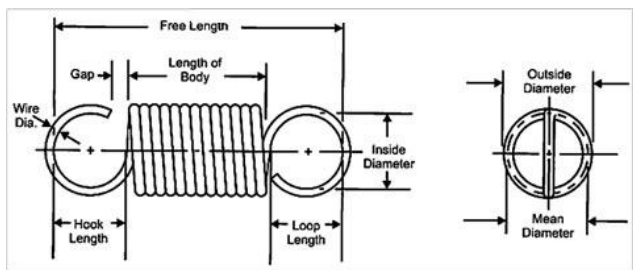


13. Which spring is stiffer (more difficult to stretch)? Explain your answer.

Part 4, Buying Springs:

The spring you used in this activity was selected to have a low spring constant, so you could make relatively large changes in its length with relatively small (and safe) forces. It was also suspended with an initial stretch so you could see how the length changes under both extension and compression. Many millions of springs are produced and sold every year, each with specific characteristics which match the intended use. *Hardware Products Company* in Chelsea, Massachusetts has been a major manufacturer since 1866 when it began producing gears, couplings, pulleys, springs and other hardware for the New England textile industry. As the demand for textile machinery moved south and abroad, the company's focus turned to high-quality springs of all types. They currently stock 3500 different springs, in addition to offering custom manufacture of countless others.

The table below from *Hardware Products'* current online catalog shows their off-the-shelf selection of 1-inch diameter extension springs. These are springs that are designed only to be stretched, not compressed. As shown in the diagram, the springs are wound such that the coils do not begin to stretch apart until the applied force reaches the "Initial tension." (Table and diagram used with permission of *Hardware Products Company*.)



Item #	Initial Tension (pounds)	Maximum Extension (inches)	Maximum Load (pounds)	Spring Rate (lbs./inch)	Alloy	Unit Price
22093	2.0	2.4	15.3	5.5	Carbon Steel	$ 5.18
22093S	2.0	2.4	13.3	4.7	Stainless Steel	$ 5.31
22094	3.0	2.0	19.5	8.3	Carbon Steel	$ 5.31
22094S	3.0	2.0	17.2	7.1	Stainless Steel	$ 5.44
22095	4.5	1.5	25.6	13.7	Carbon Steel	$ 5.31
22095S	4.5	1.5	21.9	11.6	Stainless Steel	$ 5.44
22096	14.0	0.6	56.0	68	Carbon Steel	$ 5.44
22096S	14.0	0.6	50.0	58	Stainless Steel	$ 5.57
22097	60.0	0.2	162.0	600	Carbon Steel	$ 5.57

14. What we called the "spring constant," k, is shown in this table as "Spring Rate." Explain why the spring constant can be considered a "rate."

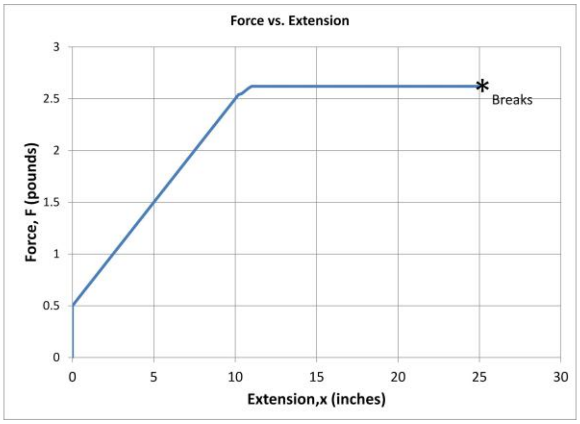
15. Based on the data in the table, which material is more flexible, carbon steel or stainless steel? Justify your answer.

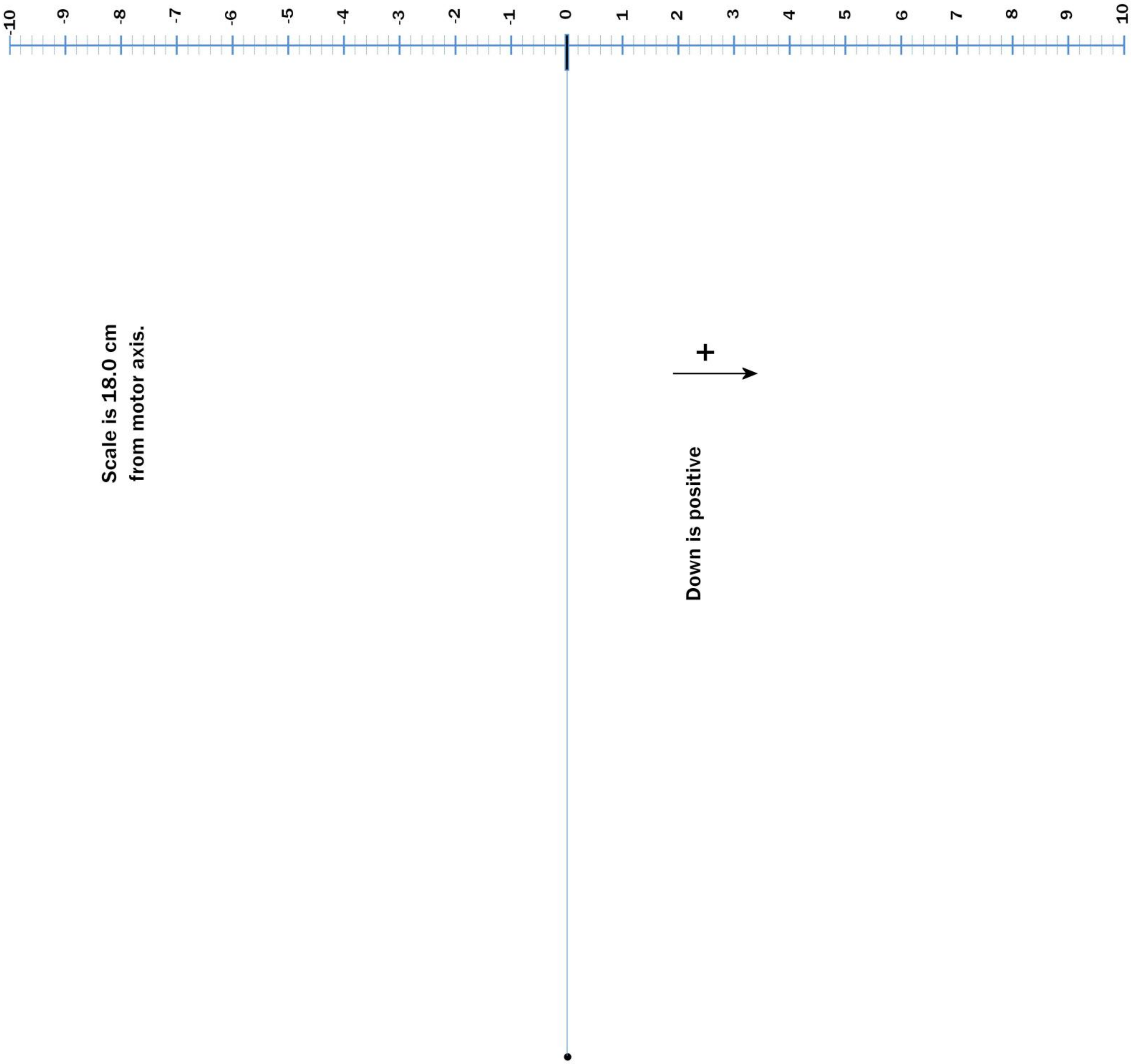
16. The basic form of Hooke's Law, F = kx, indicates that F divided by x should yield the spring constant, k. However, dividing the maximum load by the maximum extension for springs in the table above does NOT yield the "rate" as shown in the table. Explain why not.

17. In what ways is Item #22097 different from all of the other springs in the table?

18. One of the largest off-the-shelf compression springs available from *Hardware Products* is Item 21002 which has an outside diameter of 6 inches, a "free length" of 6 inches, a maximum compression of 1.4 inches, and a rate (spring constant) of 3500 lbs./inch. What is the maximum load for this spring? Show your work.

19. The graph at right shows force vs. extension for a spring such as those in the table above when the spring is stretched well beyond the maximum for which it was designed. Based on the graph, identify the domain (values for x) and range (values for F) over which Hooke's Law, $\Delta F = k \, \Delta x$, is valid.





Hooke's Law

Centimenter Scale
Center Zero
Positive Down

Function Plane

Learning with Math Machines
Englewood, OH 45322
www.mathmachines.net