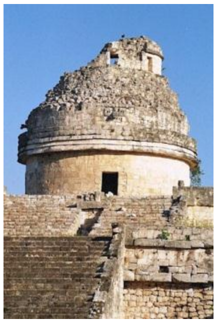


Mayan Observatory, Built approx. 900 AD
(Photo by Fred Thomas)

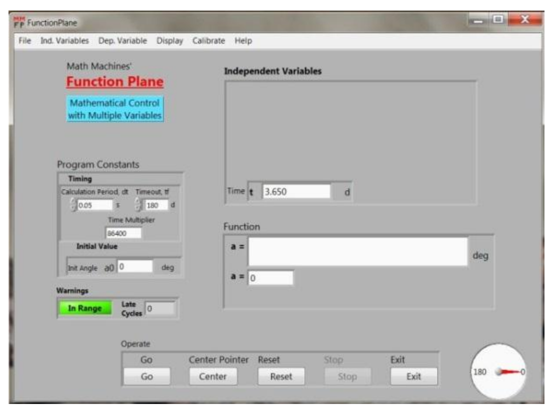
History of Science and Technology

Before there were streetlights, TV and automobiles, virtually everyone was familiar with the motions of the Sun, Moon, stars and brightest planets. Peoples all over the world used these motions to assist in timing their activities. The apparent motion of the Sun has always been important to decisions such as knowing when to start walking home to arrive safely before dark or knowing when to plant crops to take advantage of rains or warm weather. The motion of the Moon is still the basis for timing important holidays including Christian Easter, Jewish Rosh Hashanah, Islamic Eid ul-Fitr and Chinese New Year.

Prague Astronomical Clock, Built 1410 AD
(Photo by Krzysiu "Jarzyna" Szymański)

Although sundials and sighting instruments could provide reasonably precise measurements and predictions of celestial motions, a major step forward in the history of modern science and technology occurred when clocks and calendars began to replace direct observations of the sky. Early predecessors of modern clocks include mechanical devices built by Chinese monk and mathematician Yixing around 720 and by French architect Villard de Honnecourt around 1250. The oldest example still in operation was installed in Prague in 1410 by clockmaker Mikuláš of Kadaň and professor of mathematics and astronomy Jan Šindel. All three of these clocks were intended to replicate the apparent motion of the Sun, Moon and other celestial objects as well as recording the passage of time during the day.

Task: In this activity you will take on the same task as Yixing, de Honnecourt, Mikuláš and Šindel, but using *mathematical functions* rather than mechanical gears, shafts, locks and pins. You will begin by orienting the Function Plane to match the plane of the Earth's orbit around the Sun. You will then use ratios to determine the average rates at which the Sun and the Moon move against the background stars of the sky, and you will use those rates to construct mathematical functions which predict future positions of the Sun and the Moon.



Setup: For this activity, the Function Plane's servo motor must be programmed for lower-resolution operation that produces a full 180 degrees of motion. The motor can be programmed using Hitec's HFP-20 Digital Servo Programmer.

Additional Materials: 2 constellation overlays ("Western Zodiac, Aries to Virgo" and "Chinese Lunar Mansions, Wall to Wings"), wooden support, protractor, directional compass.

Math Machines Program:

Function Plane

Activity File: CelestialMechanics1

Part 1, Orienting the Function Plane:

The diagram at right shows the position of the Earth in its orbit at the time of the Vernal Equinox.1 On this date (about March 20th) the Sun is aligned with the Earth's equator, marking the beginning of spring in the northern hemisphere. Although the Earth is actually revolving counterclockwise around the Sun, it appears to us on Earth that the Sun is revolving counterclockwise around us. In 365.25 days, the Sun will appear to travel 360° through the 12 signs of the Zodiac before returning to its original position against the background stars.

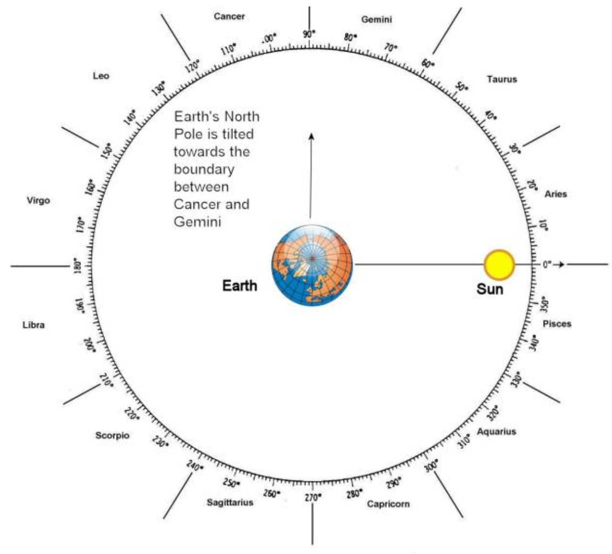


Figure 1: Earth-Centered View on the Day of the Vernal Equinox (Looking down from the north at the plane of the Solar System)

1. Place the Function Plane on a table so (a) it is horizontal and (b) the front of the Function Plane points south. Directly on Figure 2, draw a small box to show how your Function Plane is aligned with your local horizon. (Don't worry yet about your exact location on the globe. Use the line on the diagram labeled "Local Horizon" which represents sunset, a location just moving from daylight to dark.)

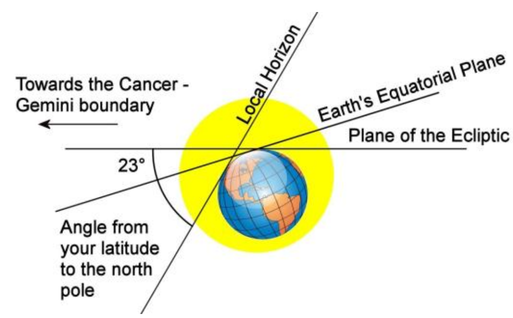


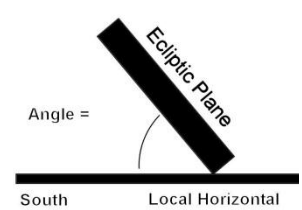
Figure 2: Earth's Tilt at the Vernal Equinox (Looking from the Virgo-Libra boundary towards the Earth with the Sun beyond)

2. Imagine you are carrying the Function Plane northward, travelling over the horizon until you reach the North Pole. On this trip, the front (south) end of the Function Plane would tilt upwards by 1° for every degree of latitude you move north. Once you reach the North Pole (even theoretically) the Function Plane will be parallel with the Earth's Equatorial Plane. In the space below record your current latitude[2] and the angle through which you would move before reaching the North Pole (latitude 90°).

   Your latitude: ____________ degrees

   Angular distance to the North Pole: ___________degrees

3. Since the Earth's equatorial plane is tilted by 23° from the plane of the Earth's orbit, you need to travel an addition 23° in your theoretical journy before reaching a point where the local horizon would be parallel with the ecliptic plane. Increase the tilt of your Function Plane so it is parallel with the ecliptic plane, and show that angle in the diagram at right.



[1] This terminology applies only to the Northern Hemisphere, since this date marks the beginning of **autumn** south of the equator.
[2] One place to find your latitude is at www.webmath.com.

Part 2, Using Ratios to Find Rates of Motion:

At least since the time of Hipparchus in the 2nd century BC, some sky observers have used angular measures to record and predict the positions of the Sun, Moon and planets. The zero position is defined as the Sun's apparent location among the background stars at the time of the Vernal Equinox—the day when the Sun crosses the Earth's equator moving north. Ancient Western astrologers designated this point as the beginning of the astrological sign, Aries. They divided the full 360° circle of the ecliptic into 12 signs, each 30° wide and each named for a nearby constellation.[3] The Sun, Moon and planets travel at different rates along nearly the same path in the sky, all passing through the same set of constellations before they return again to the Vernal Equinox.

It takes one year (actually 365.25 days) for the Sun to make a complete trip through the constellations. The Moon follows a very similar path but it moves more quickly, requiring only 27.322 days before returning to the same location among the stars.[4] By tradition, astronomers often measure the angular positions using the old units of arc-hours, arc-minutes and arc-seconds, but it is more convenient for this activity to measure the positions with the standard modern unit of degrees. Measured in degrees, a compete revolution of the Sun, Moon or a planet is a full circle, or 360°. The Sun and Moon and (usually) the planets move eastward among the stars, and that is taken to be the positive direction.

4. Use the data above to calculate the average rate at which the Sun and the Moon move eastward against the background stars, giving your answers in units of degrees/day. Express each answer using 5 significant figures and show your method clearly.

Sun's average rate = _________ degrees/day

Moon's average rate = _________ degrees/day

[3] Precession has shifted the Earth's orientation since the astrological signs were established, so the astrological signs no longer match the actual position of the constellations.

[4] This is the "sidereal month," measuring the Moon's position against the background stars. The "synodic month" is 29.531 days, the time required for the Moon to return to its same position with respect to the Sun. The synodic month is a little longer than the sidereal month, since the Moon has to travel a little extra distance to "catch up" with the Earth in its orbit around the Sun.

Part 3, Using Rates to Build a Mathematical Model of the Sun's Motion:

5. On or around March 20th, the Sun will be located at the Vernal Equinox, where its right ascension while be "a = 0°." Explain in words below how you could use the average rate you found above to predict the Sun's location (in degrees) on any date between March 20th and the first day of autumn (around September 22nd).

6. Translate the method you described above in words into a mathematical function, a = f(t), where "a" represents the Sun's angular position (in degrees) and "t" represents the number of days after the Vernal Equinox.

   a = ___________________________

7. In our model, the Pointer Plane represents the plane of the ecliptic. Select one of the available protractors which can show the Sun's movement during the six-month period which begins on March 20th and which also shows the signs of the Western zodiac. Which protractor did you select and why?

Run the computer program, "Function Plane" and load the file, "CelestialMechanics1". Type your formula from question 6 above into the box after "a =". Note that any multiplication must be entered using the symbol "*". Click "Go" and observe the motion of the Sun in your model. The simulation runs 86,400 times faster than the real motion, so 1 day only needs 1 second of computer time. Repeat the motion as you answer the questions below.

8. Using your model, how many days after the Vernal Equinox will the Sun enter the region of the sky which astrologers call Gemini?

9. Using your model, how many days after the Vernal Equinox will the Sun enter the region of the sky which astrologers call Leo?

10. As used in this simulation with the Function Plane, the domain for the function you developed in answer to question 6 is all the values for time, t, which would yield valid results. State the domain for your function in terms of the maximum and minimum possible values for t which are valid in the simulation.

11. As used in this simulation with the Function Plane, the range for the function you developed in answer to question 6 is all the values for angle, a, which could result. State the range for your function in terms of the maximum and minimum possible values which are valid in the simulation.

12. Replace the Western zodiac protractor with the equivalent protractor showing the "Chinese lunar mansions," and run your program again. Why does the same mathematical function work for both the Western and Chinese constellations?

13. How many days are required for the Sun to move from the Vernal Equinox to the "Hairy Head"?

14. What differences do you notice between the Western and Chinese systems for identifying regions in the sky along the pathway of the Sun and Moon?

15. CHALLENGE QUESTION: Using the average rate of the Sun's motion is strictly valid only if the Sun moves at constant speed. While Copernicus, Galileo and many other scientists were convinced that the Sun did move at constant speed, Kepler recognized that the Earth moves in a slightly elliptical orbit and that its speed is not quite constant. When the Earth is closest to the Sun around January 3rd, the angular rate of motion is about 3% higher than the average. When the Earth is furthest from the Sun around July 4th, the angular rate of motion is about 3% lower than average. If the model used here were modified to take the elliptical nature of the Earth's orbit into account, would this increase or decrease the answer to question 8 above about the number of days needed for the Sun to reach Leo? Approximately how much difference would there be in terms of the number of days?

Part 4, Using Rates to Build a Mathematical Model of the Moon's Motion:

16. On November 3rd, 2014 the Moon will be located at the Vernal Equinox, where its position while be "a = 0°." Explain in words below how you could use the average rate you found in Part 2 to predict the Moon's location (in degrees) on any date during the two weeks after this date.

17. Why is it necessary to know both the Moon's position on a specific date and the rate of motion you found earlier in order to make this calculation?

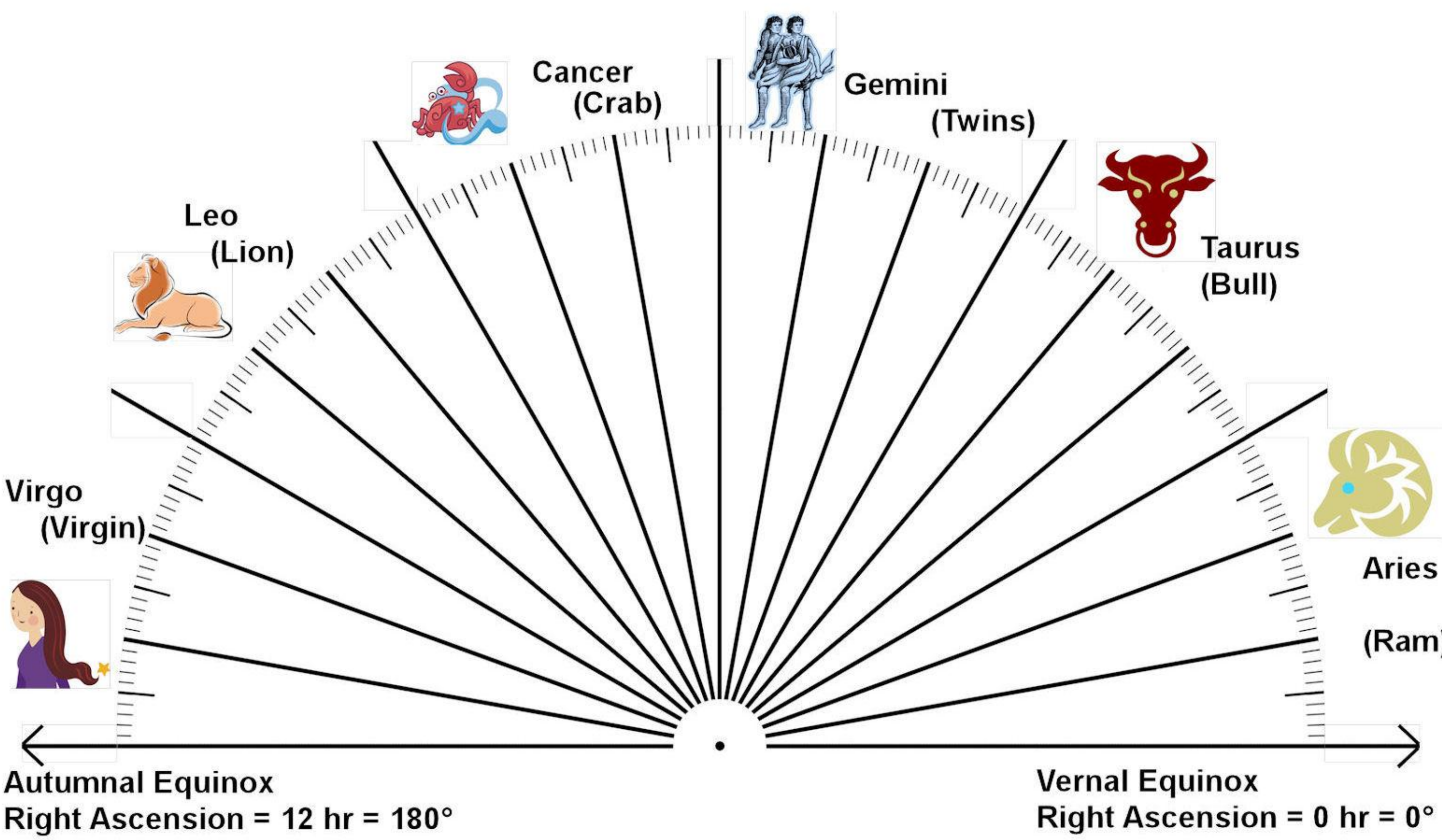
18. Translate the method you described above in words into a mathematical function, a = f(t), where "a" represents the Moon's angular position (in degrees) and "t" represents the number of days after November 3rd, 2014.

    a = ___________________________

Again use the computer program, "Function Plane," the file, "Celestial Motions," and the protractor with the Western zodiac. Type your formula from question 17 above into the box after "a =". Click "Go" and observe the motion of the Moon in your model. Repeat the motion to answer the questions below.

19. Using your model, how many days after November 3rd, 2014 will the Moon reach the region of the sky which Western astrologers call Virgo?

20. Using your model, how many days after November 3rd, 2014 will the Moon enter the region of the sky which Chinese astrologers call the "Well"?



Protractor

Center zero
Degrees
Western Zodiac
(Aries to Virgo)

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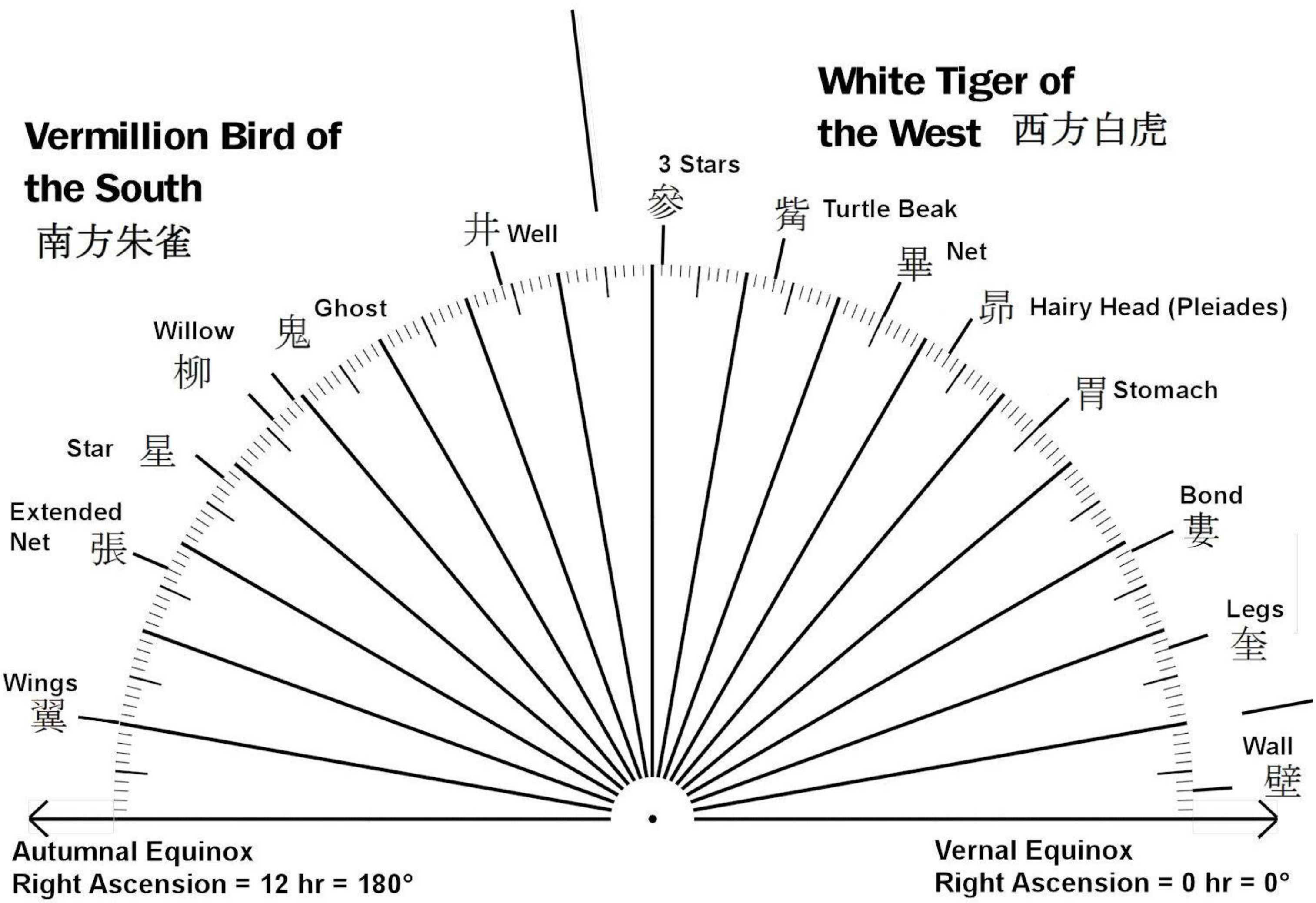

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Protractor

Degrees

Chinese Lunar Mansions
(Wall to Wings)